formed on the lateral part of the body can shield the surface not only partially but completely from the action of high-velocity particles. The resulting amount of erosion damage of the surface is determined by the initial period of formation of the dust layer with a characteristic time for the process of $\Delta t_{er} \sim 1/2pk_c$.

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CALCULATION OF THE VIRTUAL MASS OF SPHERICAL PARTICLES IN A DISPERSED MEDIUM

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One of the central problems in the mechanics of multiphase dispersed media is the problem of determining the interphase interaction. This problem is most simply solved by using relations which are valid for a single inclusion, moving in an unbounded carrying medium. This approach, however, does not take into account the effect of the inclusions on one another through the carrying medium, which can lead to considerable errors in the determination of the interphase interaction [1, 2].

The effect of inclusions on one another is easy to take into account within the framework of the cellular approach, which is analyzed in [1]. The method of cells is applicable to the study of media with a regular structure. Dispersed media, as shown in [1], have different microstructures under different conditions: a regular structure, when the distance between neighboring inclusions is the same; a chaotic structure, when the inclusions are distributed randomly, and others.

In the general case, the average interphase interaction force is found by averaging the interphase interaction force over all positions of the inclusions. However, the distribution function of the positions of the inclusions depends on the interphase interaction force. Thus, to determine the average interphase interaction, it is necessary to solve a very complicated problem.

Except for the rare exception [3], in solving this problem it is assumed that a dispersed medium has either a regular or chaotic structure. However, even after this assumption is made, it is difficult to determine the average interphase interaction, since it is difficult to determine the interphase interaction for a specific distribution of inclusions. For this reason, many authors use different simplifying assumptions in calculating the average interphase interaction [4-6]; in addition, within the framework of their approach, it is impossible to estimate the error introduced by these assumptions. The results obtained using the exact solution of the problem of interaction of several inclusions in the carrier medium are more reliable [7, 8].

In this paper, we examine the problem of the motion of spherical inclusions in an ideal carrier medium. We describe the technique for calculating the average characteristics of the interaction of inclusions with the carrier medium. This technique is used to calculate the virtual mass of spherical inclusions in the dispersed medium.

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The virtual mass of inclusions in dispersed media m was previously studied in several papers. In [1], a formula is obtained, using the classical method of cells, for the virtual mass of inclusions in a medium with a regular structure: $m = (2/3)\pi R^3\rho$, where R is the radius of the inclusion and ρ is the density of the fluid. It follows from this formula that the virtual mass does not depend on the volume concentration.

A somewhat different method of cells was used in [9], where it was found that $m = (2/3)\pi R^3\rho(1 + 3\alpha_2)$ (α_2 is the volume concentration of inclusions).

This formula gives an increase in the virtual mass with an increase in the volume concentration of inclusions, which appears, at first glance, to be incorrect. However, the exact solution of the problem of the motion of two bubbles in an ideal fluid gives both a decrease and an increase in the virtual mass of bubbles depending on their relative position. These guiding considerations show that the average virtual mass can, in principle, increase with an increase in the volume concentration.

In [10], the following formula is obtained for media with a chaotic structure:

$$m = \frac{2}{3}\pi R^3\rho(1 + 2.78\alpha_2).$$

The method used in [10] is close to the method used in this work, but Prosperetti and Van Wijngaarden [10] were able to finish the calculations and to obtain a result only in the dipole approximation for the interaction force. It is evident that the virtual mass also increases with α_2.

The dispersed media with a chaotic structure were also studied in [11], where the following formula was obtained for the virtual mass:

$$m = \frac{2}{3}\pi R^3\rho\left[1 - \frac{\alpha_2}{1-\alpha_2}\left(3 + \frac{\frac{2}{3}\ln\alpha_2}{1-\alpha_2}\right)\right]\left[1 + \frac{\alpha_2}{1-\alpha_2}\left(1 + \frac{\frac{1}{3}\ln\alpha_2}{1-\alpha_2}\right)\right]^{-1}.$$

Buyevich [11] used an approximate self-consistent interaction method. It follows from the formula obtained that the virtual mass of a particle in a dispersed medium is less than the virtual mass of a single particle with $0 < \alpha_2 < 0.0025$; in addition, the virtual mass is minimum for $\alpha_2 = 0.00092$ and is equal to 99.97% of the virtual mass of a single particle. For $\alpha_2 > 0.0025$, the virtual mass of the inclusion in a dispersed medium is greater than the virtual mass of a single inclusion.

1. Basic Equations and Assumptions. The interphase interaction forces for a specific distribution of inclusions depend on the velocities of the inclusions, which in their turn depend on the interaction forces. It is very difficult to calculate the forces in the general case. For this reason, it is of interest to calculate the forces using different simplifying hypotheses. In so doing, it is natural to expect that the use of physically well-founded hypotheses will make it possible to describe qualitatively correctly the basic characteristics of the exact solution. The problem is greatly simplified if it is assumed that the velocity of the bubbles is given. For example, we can study the case of a constant velocity of bubbles, which does not depend on their distribution. However, the following scheme is probably more systematic. We shall assume that the force of interaction of inclusions with one another is small compared to the forces acting on a single bubble. In this case, the velocity of the bubbles is determined by the buoyancy force, which is the same for all bubbles, and the force of friction of the inclusion against the carrier medium. The friction force is determined by the velocity of the bubbles relative to the fluid. From here follows the fact that the velocities of the bubbles relative to the liquid for any arrangement of the bubbles are constant.

The arguments presented above are not rigorous. They should be viewed more as motivating considerations, showing the reasonableness of the hypothesis of constant relative velocity used below. We note that this hypothesis describes qualitatively correctly the basic characteristics of the motion of inclusions in the carrier medium. For example, when bubbles rise in a fluid, the hypothesis of constant relative velocity permits describing the effect of a more rapid motion of two bubbles, moving one after the other compared to the movement of a single bubble. An analogous hypothesis was successfully used in [12].

Fixing the relative velocity of inclusions permits determining the velocity field of the carrier medium with a specific distribution of inclusions. After this, the average force can be found. In performing the average, it was assumed that the inclusions are distributed "randomly," as understood in [8].

To define the concept of the velocity of inclusions relative to the carrier medium, we shall examine potential flows of the carrier medium past the inclusions. Then, the potential of the flow can be expanded in spherical harmonics in a neighborhood of any inclusion with the other inclusions distributed arbitrarily:

$$\varphi = \sum_{j=0}^{\infty} \sum_{m=0}^{j} P_j^m (\cos\theta) (\alpha_{jm} \cos m\varphi + \beta_{jm} \sin m\varphi) \left(A_{jm} r^j + \frac{B_{jm}}{r^{j+1}} \right), \tag{1.1}$$

where P_j^m are the associated Legendre functions of the first kind of order j; θ and r are the angle of the radius vector **r** (from the origin of coordinates to the point at which the potential is being determined) with the z axis and its modulus, respectively; and α_{jm}, β_{jm}, A_{jm}, B_{jm} are constants.

In the case of a single spherical particle of radius R moving with velocity $\Delta v = (\Delta v_x, \Delta v_y, \Delta v_z)$ in a stationary unbounded carrying medium, the following relations hold:

$$\alpha_{10} B_{10} = -R^3 \Delta v_z/2, \quad \alpha_{11} B_{11} = -R^3 \Delta v_x/2, \quad \beta_{11} B_{11} = -R^3 \Delta v_y/2. \tag{1.2}$$

The remaining coefficients for the motion of a single inclusion are equal to zero. In analogy to the case of a single inclusion, we shall use Eq. (1.2) as the definition of the velocity of relative motion of a spherical inclusion in a dispersed medium with respect to the known coefficients of the expansion (1.1).

We shall study the flow of a dispersed medium in a region G with boundary Γ_0. The potential of the flow of an ideal fluid φ past spherical inclusions satisfies the equations

$$\Delta\varphi = 0 \quad \text{for} \quad r \in G$$

and the boundary conditions

$$\partial\varphi/\partial n|_{\Gamma_0} = \mathbf{n} \cdot \mathbf{v}_0(\Gamma_0), \quad \partial\varphi/\partial n|_{\Gamma_i} = \mathbf{n} \cdot \mathbf{v}_i,$$

where **n** is the normal to the boundary; $\mathbf{v}_0$ is the velocity at the boundary of the region; Γ_i is the boundary of the i-th bubble; and $\mathbf{v}_i$ is the velocity of the i-th inclusion. The vectors $\mathbf{v}_i$ are chosen so that the velocity of the inclusions relative to the fluid would equal $\Delta\mathbf{v}(\mathbf{r})$ ($\Delta\mathbf{v}(\mathbf{r})$ is a given smooth function of the coordinates).

We shall construct the potential of the flow of the fluid φ by means of successive approximations, reflecting perturbations from some inclusions relative to others [13].

The first approximation φ^1 is sought in the form

$$\varphi^1 = \sum_{i=1}^{N} \varphi_i^1,$$

where φ_i^1 is the potential of the flow with the motion of only the i-th bubble in an unbounded liquid; φ_i^1 is found as the solution of the equation

$$\Delta\varphi_i^1 = 0, \quad \varphi_i^1 \to 0 \quad \text{as} \quad |\mathbf{r} - \mathbf{r}_i| \to \infty, \quad \left.\frac{\partial\varphi_i^1}{\partial n}\right|_{\Gamma_i} = \mathbf{v}_i \cdot \mathbf{n},$$

where $\mathbf{r}_i$ is the radius vector of the center of the i-th inclusion; **r** is the radius vector of the point at which the potential is being determined. The value of $\mathbf{v}_i$ is selected from the condition that the velocity of the inclusion relative to the fluid constructed from the potential φ^1 coincide with the given velocity (Eqs. (1.1) and (1.2)).

The second approximation φ^2 is sought so that the sum $\varphi^1 + \varphi^2$ satisfies the conditions on the boundary Γ_0 of the region G occupied by the mixture:

$$\Delta\varphi^2 = 0, \quad \partial\varphi^2/\partial n|_{\Gamma_0} = \mathbf{n} \cdot \mathbf{v}_0 - \partial\varphi^1/\partial n|_{\Gamma_0}.$$

The third approximation is represented in the form

$$\varphi^3 = \sum_{i=1}^{N} \varphi_i^3,$$

where φ_i^3 is sought so that the sum $\varphi^1 + \varphi^2 + \varphi_i^3$ would satisfy the equation and the boundary condition on the surface of the i-th inclusion:

$$\Delta\varphi_i^3 = 0, \quad \varphi_i^3 \to 0 \quad \text{as} \quad |\mathbf{r} - \mathbf{r}_i| \to \infty, \quad \left.\frac{\partial\varphi_i^3}{\partial n}\right|_{\Gamma_i} = \mathbf{v}_i \cdot \mathbf{n} - \left.\frac{\partial(\varphi^1 + \varphi^2)}{\partial n}\right|_{\Gamma_i}.$$

In this case, the velocity of the inclusion $\mathbf{v}_i$ is found from the condition that the velocity of the inclusion relative to the fluid, constructed from the potential $\varphi^1 + \varphi^2 + \varphi^3$, would coincide with the given velocity (Eqs. (1.1) and (1.2)). The next approximations are found analogously.

We note that the potential of the fluid flow φ around two moving inclusions, constructed in this manner, as shown in [13], converges very rapidly to the exact potential for any (including the case of touching) distances between the inclusions.

It follows from the construction of φ that φ is represented in the form of a series whose terms either do not depend on the position of any one of the inclusions (such a term is present in φ^2) or it depends only on the position of one inclusion only (φ_i^1) or two inclusions only, etc. We single out a fixed test inclusion centered at the point $\mathbf{r}_0$, around which the external inclusions are distributed. It is convenient to sum, in analogy to [8], the terms in φ which depend on the position of precisely l external inclusions (l = 0, 1, ..., N). Then

$$\varphi(\mathbf{r} \mid \mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_N) = \sum_{l=0}^{N} \chi_l^0(\mathbf{r}), \quad \chi_0^0 = \chi_0(\mathbf{r}, \mathbf{r}_0), \tag{1.3}$$
$$\chi_l^0 = \sum_{w^N_{r_{i1},\ldots,r_{il}}} \chi_l(\mathbf{r} \mid \mathbf{r}_0, \mathbf{r}_{i1}, \ldots, \mathbf{r}_{il}),$$

where χ_l^0 depends on the position of only l external inclusions; $w^N_{r_{i1},\ldots,r_{il}}$ is a combination of l out of N inclusions (the inclusions i_1, ..., i_l are selected); $\chi_l(\mathbf{r}|\mathbf{r}_0, \mathbf{r}_{i1}, \ldots, \mathbf{r}_{il})$ depends only on the position of the inclusions i_1, ..., i_l and of the test inclusion. We shall call the quantity χ_l^0 the l-particle interaction.

The force F acting on a bubble can be easily calculated, in the case of a nonviscous fluid under study, using the equation

$$\mathbf{F} = -\int_S p d\mathbf{s}, \tag{1.4}$$

where S is the surface of the test bubble and p is the pressure in the fluid. We determine the pressure p from the Cauchy–Lagrange integral for a fluid near the test inclusion:

$$p = \rho f(t) + \rho U - \rho\partial\varphi/\partial t - \rho(\text{grad}\,\varphi)^2/2, \tag{1.5}$$

where φ is the potential of the fluid flow; ρ is the density of the fluid; t is the time; f(t) is a function of time; and U is the potential of the body force.

For simplicity, we shall restrict ourselves below to the one-dimensional formulation of the problem and we shall assume that there are no gradients of the average parameters of the medium, i.e., the velocity of the inclusions relative to the fluid is oriented along the z axis and all average characteristics of the flow do not depend on the coordinates.

We substitute (1.5) into (1.4). The first term in (1.5) gives a zero contribution to the force F and the second term gives the buoyancy (Archimedes) force F_A. We average the equation obtained over the position of all bubbles, except that of the test bubble, i.e., we multiply by the distribution function and integrate over the space of states. The final expression for the force averaged over the position of the centers of other inclusions <F>, acting on the trial inclusion in a one-dimensional flow (along the z axis), has the form

$$\langle F\rangle = F_A + \rho\left\langle\int_S \frac{\partial\varphi}{\partial t}\cos\theta ds\right\rangle + \frac{\rho}{2}\left\langle\int_S (\operatorname{grad}\varphi)^2\cos\theta ds\right\rangle, \tag{1.6}$$
$$F_A = -\frac{4}{3}\pi R^3\rho g,$$

where g is the acceleration of gravity. It can be shown that the third term in (1.6), in the case that there are no gradients of the average characteristics of the flow, vanishes. For this reason, we shall examine below only the second term, which, substituting (1.3), can be represented in the form

$$\left\langle\int_S \frac{\partial\varphi}{\partial t}\cos\theta ds\right\rangle = \sum_{l=0}^{N}(\alpha_2)^l\frac{1}{l!}\left(\frac{3}{4\pi R^3}\right)^l\frac{1}{V_0}\int\int_{\mathbf{r}_1,\ldots,\mathbf{r}_l}\int_S\frac{\partial}{\partial t}\chi_l(\mathbf{r}\,|\,\mathbf{r}_0, \mathbf{r}_1, \ldots \tag{1.7}$$
$$\ldots, \mathbf{r}_l)\, f_{l+1}(\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_l)\cos\theta ds d\mathbf{r}_1\ldots d\mathbf{r}_l,$$

where f_{l+1} is the $(l + 1)$-particle distribution function of the centers of the bubbles; V_0 is the volume of the mixture.

The sum (1.7) is the correct expansion of the starting integral in powers of α_2 in the case when the integrals on the right side of (1.7) have finite limits as $V_0 \to \infty$ and $\alpha_2 \to 0$. It is shown in [8] that for media with a regular structure, this is not the case.

2. Proof of the Correctness of the Expansion (1.7) for $l \geqslant 2$. The quantity χ_l can be represented in the form of a sum of two terms: The first term χ_l^b is boundary-independent and the second term χ_l^Γ appears as a result of the boundary $\chi_l = \chi_l^b + \chi_l^\Gamma$.

We shall study the structure of χ_l^b. To assess the convergence of the integrals in (1.7), it is important to know the behavior of χ_l^b far from the centers of the inclusions $\mathbf{r}_0$, $\mathbf{r}_1$, ..., $\mathbf{r}_l$. For this, it is convenient to use the expansion of χ_l^b in spherical harmonics. In the analysis of the convergence, only the leading order spherical harmonics, which drop off most slowly, need be retained. Every odd approximation after the first one (third, fifth, etc.) changes the coefficients of all spherical harmonics in the expansion except the zeroth one (the coefficient in front of which is equal to zero) and, as shown above, the first harmonic (the coefficient in front of which is determined by the velocity of relative motion and remains constant (1.1) and (1.2)). For this reason, the leading order spherical harmonics χ_l^b (for $l \geqslant 2$) will be quadrupoles. Quadrupoles for $r \to \infty$ drop off as $1/r^3$. It can be shown that with the reflection of a dipole or quadrupole of intensity q in the i-th inclusion, a quadrupole of intensity $q\left(\frac{R}{|\mathbf{r}_i - \mathbf{r}_j|}\right)^4$ or $q\left(\frac{R}{|\mathbf{r}_i - \mathbf{r}_j|}\right)^5$, respectively, appears. Then, the following estimate holds for the quantity χ_l^b:

$$\chi_l^b = k\left(\frac{R}{|\mathbf{r}_1 - \mathbf{r}_2|}\right)^4\left(\frac{R}{|\mathbf{r}_2 - \mathbf{r}_3|}\right)^5\left(\frac{R}{|\mathbf{r}_3 - \mathbf{r}_4|}\right)^5\cdots\left(\frac{R}{|\mathbf{r}_{l-1} - \mathbf{r}_l|}\right)^5\frac{1}{|\mathbf{r}_l - \mathbf{r}|^3},$$

where the first factor describes the intensity of the quadrupole in the second inclusion, induced by the dipole in the first one; the last factor describes the magnitude of the potential induced at the point r by the quadrupole in the l-th inclusion; and the remaining factors describe the intensity of the quadrupoles in the i-th inclusion, induced by the quadrupole in the (i − 1)-th inclusion. The quantity k depends on the velocities of the bubbles and the cosines of the angles between the vectors $\mathbf{r}_i - \mathbf{r}_j$, $\mathbf{r}_l - \mathbf{r}$ and the coordinate axes. Then, taking into account the fact that in the calculation the term $\nabla\partial\varphi/\partial t$ is important, it can be shown that if finite accelerations (of the fluid and of the bubbles) are realized, then the following estimate is valid:

$$\int_S \frac{\partial\chi_l^b}{\partial t}\cos\theta ds \leqslant R^2\frac{R}{|\mathbf{r}_l|}k_1\left(\frac{R}{|\mathbf{r}_1 - \mathbf{r}_2|}\right)^4\left(\frac{R}{|\mathbf{r}_2 - \mathbf{r}_3|}\right)^5\cdots\left(\frac{R}{|\mathbf{r}_{l-1} - \mathbf{r}_l|}\right)^5\frac{1}{|\mathbf{r}_l|^4}, \tag{2.1}$$

where k_1 is a constant for the entire volume V_0.

The integrals in the expansion (1.7) depend on the behavior of $(l + 1)$-particle distribution function $f_{l+1}(r_0, r_1, \ldots, r_l)$. The obvious properties of the function $f_{l+1}(\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_l)$ for a chaotic distribution of the inclusions $(f_{l+1}(\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_l) < C$, $f_{l+1}(\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_l) = 0$ for $|r_i - r_j| < 2R)$, taking into account (2.1), finally yield

$$\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_l} \int\limits_S \frac{\partial \chi_l^b}{\partial t} \cos\theta ds f_{l+1}(\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_l)\, d^3\mathbf{r}_1 \ldots d^3\mathbf{r}_l < c_1, \tag{2.2}$$

where c_1 is a constant which does not depend on α_2 and V_0, which proves the validity of the expansion (1.7).

The effect of the boundary of the region is studied analogously. For a flat boundary, the function χ_l^Γ is constructed explicitly by the method of images relative to the surface of the boundary. The corresponding estimates are easily made analogously to the case of the absence of boundaries. For an arbitrary region, it is more difficult to take the boundary into account, but it can be shown that, for example, estimates of the type (2.2) are also valid for convex regions V_0.

The calculations and arguments presented above prove that the sum (1.7) is the correct expansion of $\left\langle \int\limits_S \frac{\partial \varphi}{\partial t} \cos\theta ds \right\rangle$ in powers of $(\alpha_2)^l$ for $l \geqslant 2$. This means that in order to determine the average of the force with accuracy up to α_2 it is sufficient to study the first two terms in (1.7).

3. Calculation of the Contribution of the First Two Approximations of φ to the Interphase Interaction Force. To calculate the first two terms on the right side of (1.7), it is necessary to use all of the approximations of the iteration method in constructing the potential φ. It is shown below that it is very difficult to calculate the contribution from the first two approximations φ_1 and φ_2 directly from Eq. (1.7). For this reason, the contribution from φ_1 and φ_2 is calculated using a special device. Successive approximations of φ are taken into account directly according to Eq. (1.7).

The following equation is obtained from the construction of φ_1 and φ_2:

$$\frac{\partial(\varphi^1 + \varphi^2)}{\partial t} = -\sum_{i=1}^{N} \left[\frac{R^3}{2} \frac{\partial \Delta v(\mathbf{r}_i)}{\partial t} \frac{\cos\theta}{|\mathbf{r}_i - \mathbf{r}|^2} + \frac{\Delta v(\mathbf{r}_i) R^3}{2} (v(\mathbf{r}_i) - v(0)) \frac{3\cos^2\theta - 1}{|\mathbf{r}_i - \mathbf{r}|^3} \right] + \frac{\partial \varphi^2}{\partial t}, \tag{3.1}$$

where θ is the angle between the vector $\mathbf{r} - \mathbf{r}_i$ and the z axis and $v(\mathbf{r}_i)$ is the velocity of the i-th inclusion.

Substitution of the separate terms on the right side of (3.1) into (1.7) leads to integrals which diverge in the limit $V_0 \to \infty$. This is associated with the fact that dipoles distributed uniformly in the entire infinite volume will give rise to infinite velocities of the liquid.

Infinite velocities will not arise in real problems due to the boundary conditions, i.e., due to $\partial\varphi^2/\partial t$. This means that the integral in (1.7) in front of $(\alpha_2)^1$ over the entire expression (3.1) will be finite, though the integrals of each of the terms in (3.1) will be infinite. It is difficult to determine φ^2 with arbitrary geometry of V_0 and arbitrary boundary conditions. To circumvent this difficulty we shall relate

$$\left\langle \int\limits_S \frac{\partial(\varphi^1 + \varphi^2)}{\partial t} \cos\theta\, ds \right\rangle$$

to the average parameters of the fluid in the corresponding elementary volume. For this, we shall use Green's theorem [14], which is written in the form

$$\int\limits_V (\psi\nabla^2\Phi - \Phi\nabla^2\psi)\, dV = \int\limits_S \left(\psi \frac{\partial \Phi}{\partial n} - \Phi \frac{\partial \psi}{\partial n}\right) ds. \tag{3.2}$$

For the region V, we shall use the volume occupied by the mixture excluding the volume occupied by the trial inclusion. We note that V includes the volumes of the external inclusions. For ψ and Φ we shall use the functions $\partial(\varphi^1 + \varphi^2)/\partial t$ and $\cos\theta/r^2$, respectively. It follows from (3.1) that $\nabla^2\psi$ is represented in the form of a sum of delta functions and their derivatives. This permits calculating the left side of (3.2) and averaging (3.2) over the position of the external inclusions. The final formula has the form

$$\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_N}\int\limits_{S_f}\frac{\partial(\varphi^1+\varphi^2)}{\partial t}\frac{2}{R^3}\cos\theta\, ds f_{N+1}(0,\mathbf{r}_1,\ldots,\mathbf{r}_N)\, d^3\mathbf{r}_1\ldots d^3\mathbf{r}_N = \tag{3.3}$$

$$= -\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_N}\int\limits_{S_f}\frac{\cos\theta}{R^2}\frac{\partial}{\partial r}\left[\frac{\partial(\varphi^1+\varphi^2)}{\partial t}\right] ds f_{N+1}(0,\mathbf{r}_1,\ldots,\mathbf{r}_N)\, d^3\mathbf{r}_1\ldots d^3\mathbf{r}_N -$$

$$-\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_N}\int\limits_{\Gamma_0}\left[\psi\frac{\partial}{\partial n}\frac{\cos\theta}{r^2} - \frac{\cos\theta}{r^2}\frac{\partial\psi}{\partial n}\right] ds f_{N+1}(0,\mathbf{r}_1,\ldots,\mathbf{r}_N)\, d^3\mathbf{r}_1\ldots d^3\mathbf{r}_N +$$

$$+\frac{4\pi R^3}{2}\int\limits_{\mathbf{r}_1}\frac{3\cos^2\theta_1 - 1}{(r_1)^3}\frac{\partial\Delta v(\mathbf{r}_1)}{\partial t} ng(0, r_1)\, d^3\mathbf{r}_1 - \frac{4\pi R^3}{2}\int\limits_{\mathbf{r}_1}\frac{9\cos\theta_1 - 15\cos^3\theta_1}{(r_1)^4}\times$$

$$\times\Delta v(\mathbf{r}_1)[v(\mathbf{r}_1) - v(0)]\, ng(0, r_1)\, d^3\mathbf{r}_1,$$

where g is the binary correlation function for a chaotic distribution of inclusions [15]; θ_1, r_1 are the coordinates of the vector $\mathbf{r}_1$; and S_f is the surface of the trial bubble. Equation (3.3) permits calculating the contribution of φ^1 and φ^2 to the average interphase interaction force.

The first term on the right side of (3.3) can be determined by applying Green's formula (3.2), in which the volume of the trial inclusion, $\partial(\varphi^1+\varphi^2)/\partial t$, and $r\cos\theta$ are used for the volume V and the functions ψ and Φ, respectively. The remaining terms on the right side of (3.3) can be greatly simplified by applying once again Green's formula, in which the volume occupied by the mixture V_0, $\partial(\varphi^1+\varphi^2)/\partial t$, and $\cos\theta/r_2$ are used for the volume V and the functions ψ and Φ, respectively. In addition, in this case, distributions of the centers of the inclusions such that the origin of coordinates is located in the carrier medium are examined. Equation (3.3) finally assumes the form

$$\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_N}\int\limits_{S_f}\frac{\partial(\varphi^1+\varphi^2)}{\partial t}\frac{3}{R^3}\cos\theta f_{N+1}(0,\mathbf{r}_1,\ldots,\mathbf{r}_N)\, ds d^3\mathbf{r}_1\ldots d^3\mathbf{r}_N = \tag{3.4}$$

$$= -\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_N}\int\limits_{\Gamma_0}\left\{[\psi f_{N+1}(0,\mathbf{r}_1,\ldots,\mathbf{r}_N) - \psi_1 f_N(0\,|\,\mathbf{r}_1,\ldots,\mathbf{r}_N)]\frac{\partial}{\partial n}\left(\frac{\cos\theta}{r^2}\right) -\right.$$

$$\left. -\left[\frac{\partial\psi}{\partial n}f_{N+1}(0,\mathbf{r}_1,\ldots,\mathbf{r}_N) - \frac{\partial\psi_1}{\partial n}f_N(0\,|\,\mathbf{r}_1,\ldots,\mathbf{r}_N)\right]\frac{\cos\theta}{r^2}\right\} ds d^3\mathbf{r}_1\ldots d^3\mathbf{r}_N +$$

$$+\frac{4\pi R^3}{2}\int\limits_{\mathbf{r}_1}\frac{3\cos^2\theta_1 - 1}{(r_1)^3}\frac{\partial\Delta v(\mathbf{r}_1)}{\partial t} n(g - g_1)\, d^3\mathbf{r}_1 - \frac{4\pi R^3}{2}\int\limits_{\mathbf{r}_1}\frac{9\cos\theta_1 - 15\cos^3\theta_1}{(r_1)^4}\times$$

$$\times\Delta v(\mathbf{r}_1)(v(\mathbf{r}_1) - v(0))\, n(g - g_1)\, d^3\mathbf{r}_1 + 4\pi\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_N}\frac{\partial\psi_1}{\partial z}f_N(0\,|\,\mathbf{r}_1,\ldots$$

$$\ldots,\mathbf{r}_N)\, d^3\mathbf{r}_1\ldots d^3\mathbf{r}_N - 4\pi\frac{1}{2}\frac{\partial\Delta v}{\partial t},$$

where ψ and ψ_1 are equal to $\partial(\varphi^1 + \varphi^2)/\partial t$, when an inclusion is and is not present at the origin of coordinates, respectively; $f_{N+1}(0, \mathbf{r}_1, \ldots, \mathbf{r}_N)$ and $f_N(0|\mathbf{r}_1, \ldots, \mathbf{r}_N)$ are the distribution functions of inclusions when an inclusion is and is not present at the origin of coordinates, respectively; g_1 is a single-particle distribution function of inclusions under the condition that the carrier medium is located at the origin of coordinates.

By performing the quite cumbersome calculations, it can be shown that the first term on the right side of (3.4) approaches zero in the limit $V_0 \to \infty$.

In it shown in [16] that for a chaotic distribution of inclusions, with accuracy up to α_2 the functions g and g_1 can be approximated by step functions:

$$g=\begin{cases}0 & \text{for} \quad r<2R,\\ 1 & \text{for} \quad r>2R,\end{cases} \quad g_1=\begin{cases}0 & \text{for} \quad r<R,\\ 1 & \text{for} \quad r>R.\end{cases}$$

Then, retaining the first derivative in the expansion of the integrands in the second and third terms on the right side of (3.4), it can be shown that the second and third terms in (3.4) also vanish.

We relate the fourth term on the right side of (3.4) to the average acceleration of the fluid $\langle dv_l/dt \rangle$:

$$\iint\limits_{\mathbf{r}_1,\ldots,\mathbf{r}_N} \frac{\partial \psi_1}{\partial z} f_N(0|\mathbf{r}_1,\ldots,\mathbf{r}_N)\, d^3\mathbf{r}_1 \ldots d^3\mathbf{r}_N = \left\langle \frac{dv_l}{dt} \right\rangle.$$

Finally, the contribution of the first two approximations to the interphase interaction force is equal to

$$F_1=\left\langle \int\limits_S \frac{\partial(\varphi_1+\varphi_2)}{\partial t}\cos\theta ds \right\rangle = -\frac{4}{3}\pi R^3 \frac{1}{2}\frac{\partial \Delta v_l}{\partial t}+\frac{4}{3}\pi R^3 \left\langle \frac{dv_l}{dt} \right\rangle. \tag{3.5}$$

The calculations show that in the formulation under study the equality $\frac{d}{dt}\langle v_l \rangle = \left\langle \frac{dv_l}{dt} \right\rangle$ holds.

4. <u>Inclusion of Single-Particle Interaction, Arising in Higher-Order Approximations.</u> It can be shown that the boundary must be taken into account only in the second approximation, which was done in Sec. 3. The single-particle interaction coincides with the exact solution of the problem of the motion of two inclusions in an unbounded fluid. It was shown above that the higher-order approximations do not change the decreasing dipole harmonics around the inclusions. This ensures that the corrections to the potential arising in higher-order approximations with increasing distance between inclusions decrease rapidly enough for the integrals in (1.7) to converge. We construct the potential of the fluid flow in the presence of two inclusions moving in it by the method of successive approximations. The solution of the problem of the motion of a sphere in the velocity field of the source is constructed in [13]. This solution permits regarding the flow as the motion of a sphere in the field of a dipole.

Let a dipole with intensity Q, oriented along the Oz axis, be situated in the field of a dipole positioned at the origin of a spherical coordinate system. A sphere of radius R, whose center O_1 has the coordinates (θ_1, f), is located in the field of the dipole. It can then be shown that the fluid flow is a flow created by the dipole and the following singularities inside the sphere. A point dipole, with the projections $-\cos\theta_1(R/f)Q$ and $\sin\theta_1(R/f)^3Q$ on the OO_1 axis and on the axis perpendicular to OO_1, respectively, is located at the point O_2 on the OO_1 axis (see Fig. 1). The point O_2 is determined from the condition $(O_2O_1) = R^2/f$. Distributed dipoles, oriented perpendicular to the OO_1 axis with an intensity of $Q\sin\theta_1(R/f)^3\cdot(x/Rf)$, where x is the distance from the point O_1, lie along the segment O_1O_2.

The solution presented permits constructing by means of successive approximations [13] the exact solution of the problem of the motion of two inclusions.

In calculating the integrals in (1.7), the contribution from the solution constructed must be averaged over all positions of the two inclusions. After performing the averaging over the angular variables, we represent the contribution F_2 of the higher-order approximations to the interphase interaction force in the form

$$F_2=\int\limits_{2R}^{\infty} c(f)\,df.$$

For the case of large distances between the inclusions ($R/f \ll 1$), we can restrict ourselves to the first terms of the expansion of c(f) in powers of R/f, which can be calculated analytically

$$c=-\frac{20}{3}\pi\alpha_2\left(\frac{R}{f}\right)^8 R^2 \frac{d\Delta v}{dt}. \tag{4.1}$$

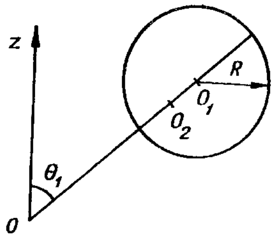

Fig. 1

For the case of small distances f, the value of c was calculated numerically. The calculations showed that for R/f > 3 the exact value of c coincides with (4.1).

Finally,

$$F_2 = -0.193\alpha_2 R^3 d\Delta v/dt. \tag{4.2}$$

We note that the use of the analytical result (4.1) only, without performing numerical calculations, merely replaces the constant 0.193 by 0.13, which indicates the good accuracy of the approximation (4.1).

Equations (3.5) and (4.2) permit obtaining a final expression for the interphase interaction force, which in an inertial coordinate system has the form

$$F = F_1 + F_2 = \rho \frac{4}{3}\pi R^3 \left(g - \left\langle \frac{dv_l}{dt} \right\rangle\right) - \frac{4}{3}\pi R^3 \rho \frac{1}{2} \frac{d\Delta v}{dt}(1 + 0.092\alpha_2). \tag{4.3}$$

The first term corresponds to the buoyancy force and the second term corresponds to the force due to the virtual mass. It is evident from Eq. (4.3) that the virtual mass of spherical inclusions in a dispersed medium is somewhat higher than the virtual mass of a single inclusion. Equation (4.3) was obtained for the case when the parameters of the mixture are uniform. The proposed method permits obtaining an expression for the force acting on an inclusion in a mixture with nonuniform parameters.

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STRUCTURE OF A LAMINAR BOUNDARY LAYER WITH DISTRIBUTED SUCTION

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Suctioning of the boundary layer for purposes of increasing the aerodynamic quality of a wing has two purposes: to make the flow laminar and to eliminate or delay its detachment. Both the study of the stability of the flow and the formulation of the variational problem of determining the energetically optimum rate of suctioning must be based on an analysis of the flow in the region near the wall, where pressure losses exist and a transition occurs from sharp changes in the velocity of a continuous (averaged) distribution. This analysis is performed within the framework of the Navier–Stokes equations with the help of the combined method of different scales and joining of asymptotic expansions for the simplest possible formulation of the problem and layout of the suction system. The conditions required for suctioning off a distributed flow of liquid, which is assumed to be given, are determined.

1. We choose as the basic unit parameters the chord of the profile, the velocity of the unperturbed flow, and the density of the fluid. In a locally Cartesian coordinate system x_1, y_1 with the x_1 axis oriented along the contour of the profile, the equation of transport of vorticity $\Delta\psi$ (ψ is the stream function, $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial y_1^2$) in a two-dimensional flow has the form $L(\Delta\psi) = 0$, where $\delta^{-2} = 1/\nu$ is the Reynolds number, ν is the coefficient of kinematic viscosity, and the quasilinear differential operator

$$L = \frac{\partial\psi}{\partial y_1}\frac{\partial}{\partial x_1} - \frac{\partial\psi}{\partial x_1}\frac{\partial}{\partial y_1} - \delta^2\Delta. \tag{1.1}$$

The rate of suctioning of the boundary layer v_{01} is equal, in order of magnitude, to the thickness of the boundary layer

$$v_{01} = \delta v_0(x_1). \tag{1.2}$$

If v_{01} is less than this quantity, then suctioning has an insignificant effect on the boundary layer. Conversely, if v_{01} is greater than $O(\delta)$, then a nonviscous flow is realized [6].

We shall assume that the suctioning is realized through a regular array of transverse slits with half-spacing $\tau = B(x_1)\delta^n$, where $0 < n \leqslant 2$. For $n > 2$, the scales of the perturbations are so small that due to the manifestation of molecular effects, the Navier–Stokes equation becomes inapplicable. The case $n \to 0$ corresponds to discrete suctioning. We shall assume that the permeability factor $x_0 = x_{01}/\tau$, where x_{01} is the half-width of a slit, is arbitrary ($0 \leqslant x_0 \leqslant 1$).

In application to suctioning of liquid through porous walls, the model of overflow of liquid, examined below, ignores the stochastic distribution of pores and their shape;

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